MIDTERM 1 (RIBET) - ANSWER KEY

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(1) $A^{-1} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 3 & -1 & -2 & 2 \end{bmatrix}$

(2) (a) **FALSE**

Take: $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then: $AB = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Which is again an elementary matrix!

(b) **FALSE**

Take A to be the zero matrix and B any nonzero vector!

(c) **FALSE**

Take $A = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Then $AB = \begin{bmatrix} 1 \end{bmatrix}$, which is invertible, even though neither A or B is invertible (they aren't even square matrices!)

(d) **TRUE** (see page 156, the null-space of an $m \times n$ matrix is always a subspace of $\mathbb{R}^n$.

(e) Ignore this. (we'll see this in chapter 4)

(f) **FALSE** (what if $\mathbf{v_1} = \mathbf{v_5}$?)

(3) $\boxed{b = 0}$. (see next page for an explanation)

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Form the augmented matrix: $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 4 & b \\ 3 & 4 & 2 \\ 4 & 5 & 3 \end{bmatrix}$. Row-reduce until you get the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 0 & 0 & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Now, in order for the third column to be in the span of the first two, we need the second row to be a row of zeros. Hence, we require $b = 0$.